WORDS TO KNOW

similar

congruent

proportional

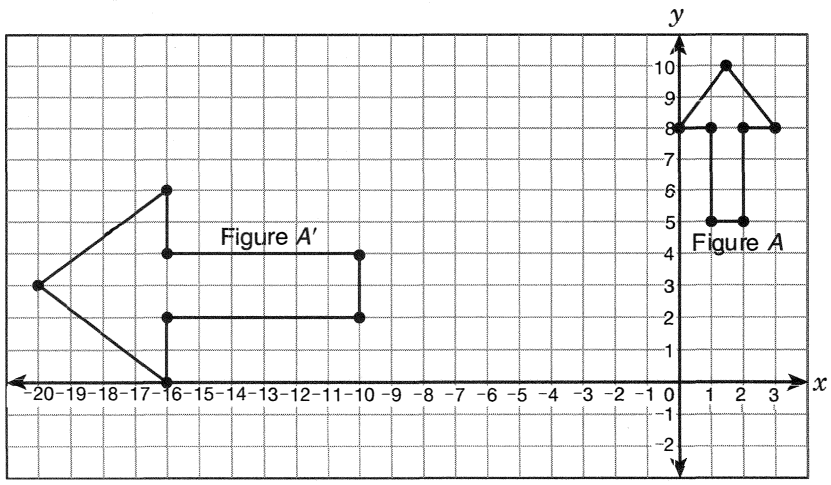
Lesson 25

SHOW SIMILARITY WITH TRANSFORMATIONS NY-8.G.4

INTRODUCTION

Real-World Connection

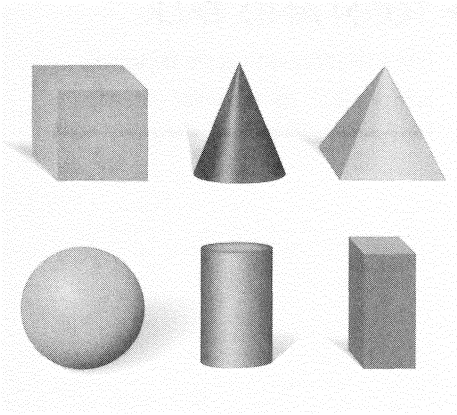
Ekani is going to paint a design on her bedroom wall. She projects a coordinate grid onto the wall to help her. If she begins with the polygon labeled Figure *A*, how can she transform the figure to create the similar Figure *A'*? Let's practice the skills in the **Guided Instruction** and **Independent Practice** and see what transformations were done at the end of the lesson!



What I Am Going to Learn

- How to transform figures in different ways
- How to use coordinates to determine whether a figure was transformed
- How to determine whether two figures are similar

What I May Already Know



- I know how to draw and describe geometric figures, and describe the relationships between them.
- I know how to draw polygons on a coordinate plane.
- I know how to show and analyze proportional relationships on a coordinate plane.

Vocabulary in Action

- Figures are **similar** if they have the same shape, but not necessarily the same size.
- If two figures are similar, their corresponding angles are **congruent**, or equal in size and shape, and the measures of their corresponding side lengths are **proportional**, or have a constant ratio. Two shapes are proportional if the ratios of their corresponding sides are the same.

Translations, reflections, and rotations are rigid transformations:

- The image is congruent to the pre-image.

A dilation is a type of transformation that changes the size of a figure by a scale factor:

- If the scale factor is > 1, the image will be larger than the pre-image. If the scale factor is between 0 and 1, the image will be smaller than the pre-image.
- A figure is dilated from a given point, usually the origin.
- The pre-image and image are similar: corresponding angles are congruent and corresponding sides are proportional.

Identifying transformations:

- If a series of transformations can be shown to map one figure on the coordinate plane to another figure on the coordinate plane, then the figures are similar.

TURN AND TALK

For a dilation with scale factor a, any point (x, y) becomes (xa, ya).

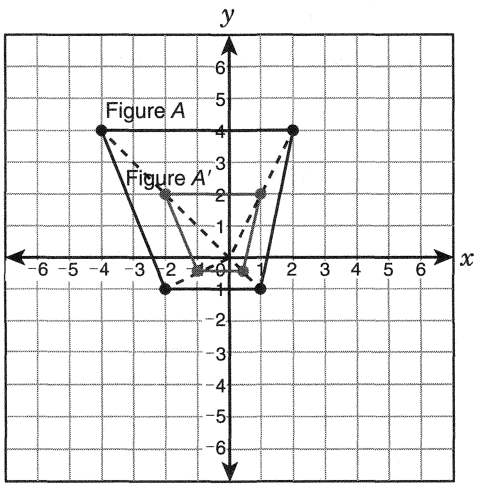
TIPS AND HINTS

The symbol ~ means "is similar to".

EXAMPLE

Is Figure A similar to Figure A'?

Each point in Figure A' is one-half the distance from the origin that it was in Figure A.



The coordinates of any vertices in the image will be one-half the value of the corresponding coordinates in the pre-image.

One of the points in Figure A is $(-4, 4)$.

Its corresponding point in Figure A' is $(-2, 2)$.

The length of each side in Figure A' is one-half the length of the corresponding side in Figure A. The top side is 6 units long in Figure A and 3 units long in Figure A'.

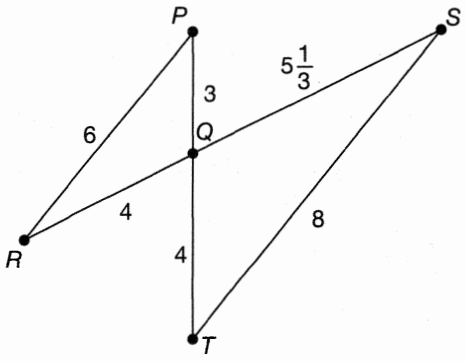
So, the figures are similar. Figure A was dilated by a scale factor of $\frac{1}{2}$.

We write:

Figure A ~ Figure A'

GUIDED INSTRUCTION

1. To design a company logo, $\angle PQR$ is transformed to create similar $\angle TQS$. Explain how the corresponding angles and sides of each triangle are related. What sequence of transformations would produce $\angle TQS$ from $\angle PQR$?



Step One Look at the corresponding angles. Write mathematical statements showing the relationship of corresponding angles of similar figures.

$\angle P = \angle T$
$\angle Q = \angle Q$
$\angle R = \angle S$

Corresponding angles of similar figures are congruent, or have the same measure.

Step Two Look at the corresponding sides. Write a mathematical statement showing the relationship between corresponding sides of similar figures.

$\frac{TQ}{PQ} = \frac{TS}{PR} = \frac{QS}{QR} = \frac{4}{3}$

Corresponding sides of similar figures are proportional. The scale factor is $\frac{4}{3}$.

Step Three Think about what you know about translations, rotations, reflections, and dilations. Identify the sequence of transformations that would produce $\triangle TQS$ from $\triangle PQR$.

Triangle TQS is created by rotating $\triangle PQR$ 180° clockwise about vertex Q. Then the image is dilated by a factor of $\frac{4}{3}$ to produce $\triangle TQS$.

TIPS AND HINTS

The symbol $\angle$ means "angle," and the symbol $\triangle$ means "triangle."

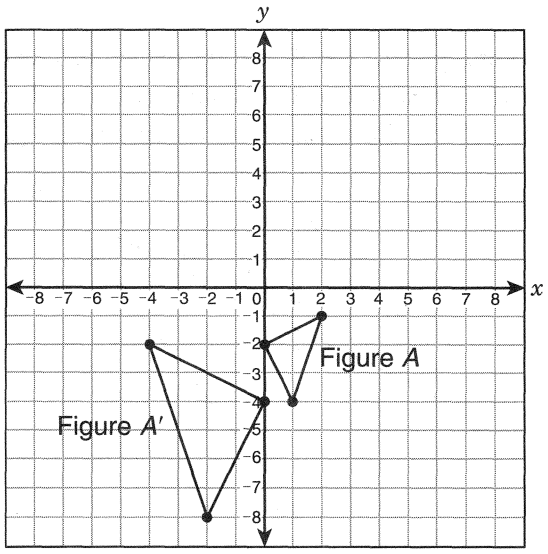
TIPS AND HINTS

One way to remember the relationships of similar figures is to imagine drawing a polygon on a balloon. Then think about what happens to the polygon as the balloon is blown up, and then some air is let out; the shape stays the same but it stretches and shrinks as the balloon changes.

TURN AND TALK

Does the order of the transformations matter? Could the reflection have been done first, then the dilation?

2. Determine if the figures are similar.



Step One Find the corresponding vertices of each triangle.

A: (2, −1), (1, −4), (0, −2)

A'(−4, −2), (−2, −8), (0, −4)

Step Two Identify the transformations that map Figure A to Figure A'. First, determine whether the two figures are similar.

The coordinates for Figure A' are twice the value of the corresponding coordinates for Figure A (ignoring signs). So, Figure A was dilated with a scale factor of [ ].

Step Three Identify additional transformations that map Figure A to Figure A'.

Find the new coordinates for Figure A when dilated by a scale factor of 2:

(4, −2), (2, −8), (0, [ ])

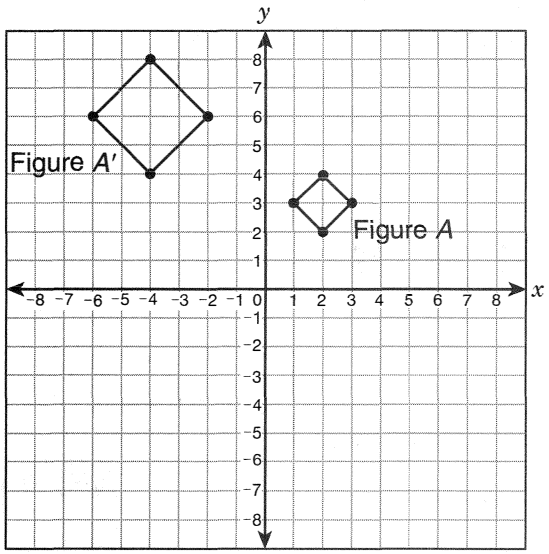
Step Four Compare the dilated Figure A coordinates to the coordinates of Figure A'.

The x-coordinates in the dilated Figure A have opposite signs to the corresponding x-coordinates in Figure A', while the y-coordinates are the same. So, the figure was reflected over the y-axis.

Step Five Decide if the figures are similar.

A dilation by a scale factor of 2 and a reflection across the y-axis mapped Figure A to Figure A'. The figures are [ ].

3. Which statement is true about Figure A and Figure A'?



TIPS AND HINTS

The image is larger than the pre-image. How does that affect how the figure was transformed?

Ⓐ Figure A can be mapped onto Figure A' by a translation of 2 units up, a reflection across the y-axis, and a dilation by a scale factor of 2.

Ⓑ Figure A can be mapped onto Figure A' by a reflection across the y-axis and a dilation by a scale factor of 2.

Ⓒ Figure A can be mapped onto Figure A' by a dilation by a scale factor of $\frac{1}{3}$, a reflection across the y-axis, and a translation of 2 units down.

Ⓓ Figure A can be mapped onto Figure A' by a reflection across the y-axis and a translation of 4 units up.

Learning Together

With a partner, use shapes and arrows to create icons representing the four different transformations: rotations, reflections, translations, and dilations. Discuss why each of the transformations results in an image that is similar to the pre-image. In your discussion, include a reference to congruent angles and both like and proportional corresponding sides.

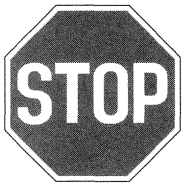
How Am I Doing?

What questions do you have?

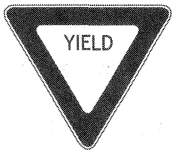
Imagine performing a dilation and translation on a figure. How are the image and pre-image similar, and how are they different?

Describe some examples of transformations (reflections, translations, and rotations, or dilations) that you would see in real-life.

Circle the sign that shows how you are doing with the skill.



I am stuck.

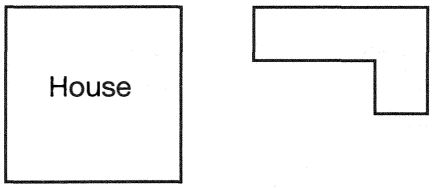


I almost have it.

I understand the skill.

INDEPENDENT PRACTICE 1

1 Jerome is buying fencing to build a dog run in his backyard as shown below.



He decides to move the dog run according to the transformation $(x, y) \rightarrow (y, -x)$. How is the cost of his project material affected?

A The cost increases.

B The cost decreases.

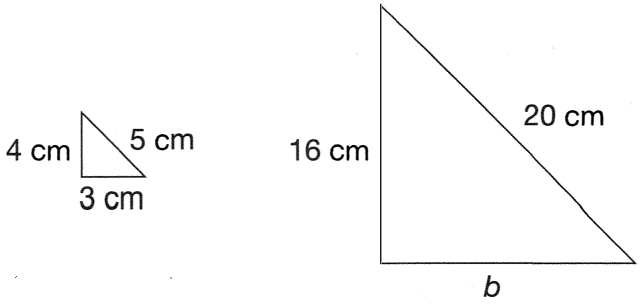
C The cost stays the same.

D The values for x and y must be known to determine the effect.

THINK ABOUT IT

Can all types of transformations make images smaller or larger?

2 Look at the two triangles below.



If the triangles are similar, what scale factor could you use to find the value of side b?

A $\frac{1}{4}$

B $\frac{1}{5}$

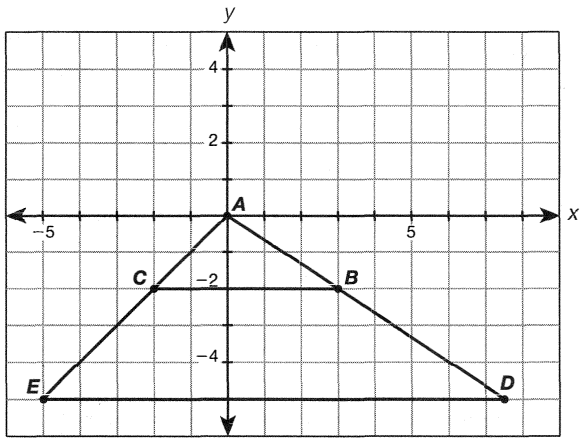
C $\frac{3}{16}$

D $\frac{4}{20}$

TIPS AND HINTS

Keep in mind that, to be similar, all sides of these two triangles must be proportional.

3 Triangles *ABC* and *ADE* are shown on the coordinate plane below.



△*ABC* and △*ADE* are similar. What type of transformation would produce △*ADE* from △*ABC*?

A a dilation with a scale factor of 0.4

B a translation 3 units down

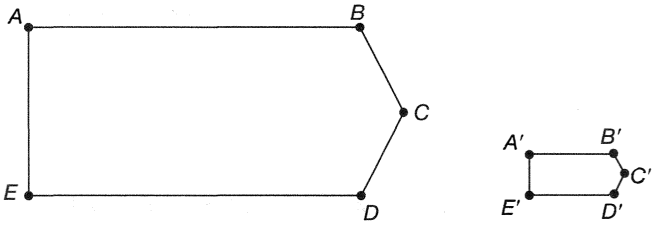
C a dilation with a scale factor of 2.5

D a reflection over the *y*-axis

TIPS AND HINTS

Make sure you are keeping the pre-image and image straight!

4 As shown below, pentagon *ABCDE* is translated right and dilated by a factor of $\frac{1}{4}$ to create pentagon *A′B′C′D′E′*.



THINK ABOUT IT

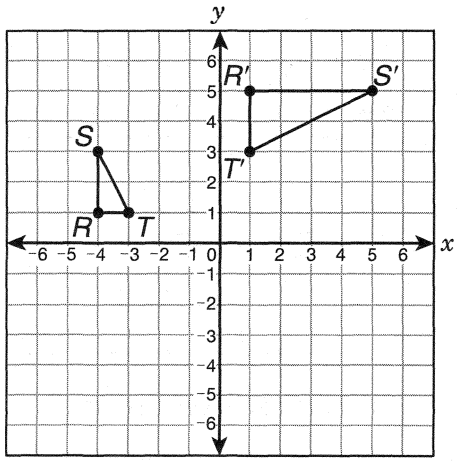
What do you have to do to change the size of an angle? What do you have to do to change the size of a line?

Which corresponding parts of the pentagons are proportional and which are congruent?

Explain your answer.

INDEPENDENT PRACTICE 2

1 Figure RST is transformed to produce the image $R'S'T'$ as shown.



Which is one transformation that was performed?

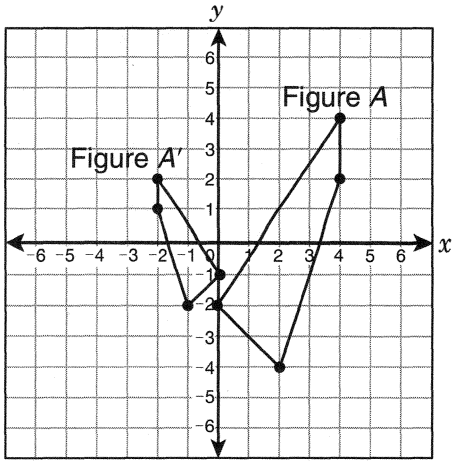
A reflection across the y-axis

B dilation by a scale factor of $\frac{1}{2}$

C rotation 90° clockwise

D dilation by a scale factor of 3

2 Select the set of transformations that are used to map Figure A onto Figure A'.



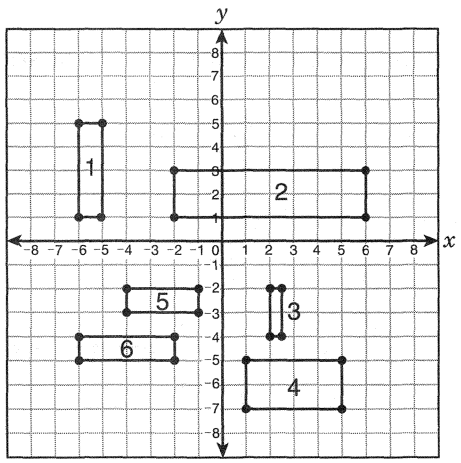
A reflection across the x-axis, rotation 45° counterclockwise

B reflection across the y-axis, dilation by a scale factor of $\frac{1}{2}$

C dilation by a scale factor of $\frac{1}{2}$, rotation 45° counterclockwise

D rotation 45° counterclockwise, dilation by a scale factor of 2

3 Look at the six rectangles on the grid below.



Which statement is true about the rectangles on the grid?

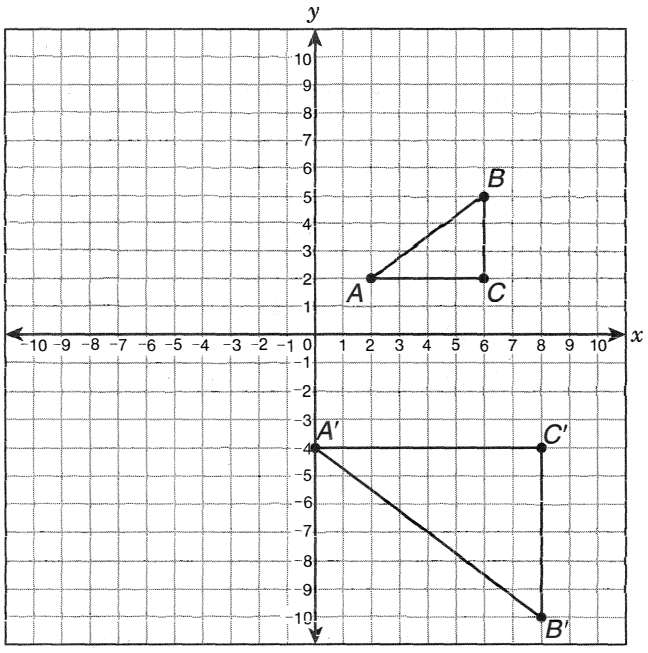
A Rectangle 1 is similar to rectangle 2.

B Rectangle 5 is similar to rectangle 4.

C Rectangle 3 is similar to rectangle 5.

D Rectangle 4 is similar to rectangle 2.

4 As shown below, triangle ABC is mapped onto triangle $A'B'C'$.



First, the image is reflected across the x-axis, then it is translated left 2 units. What is the last transformation?

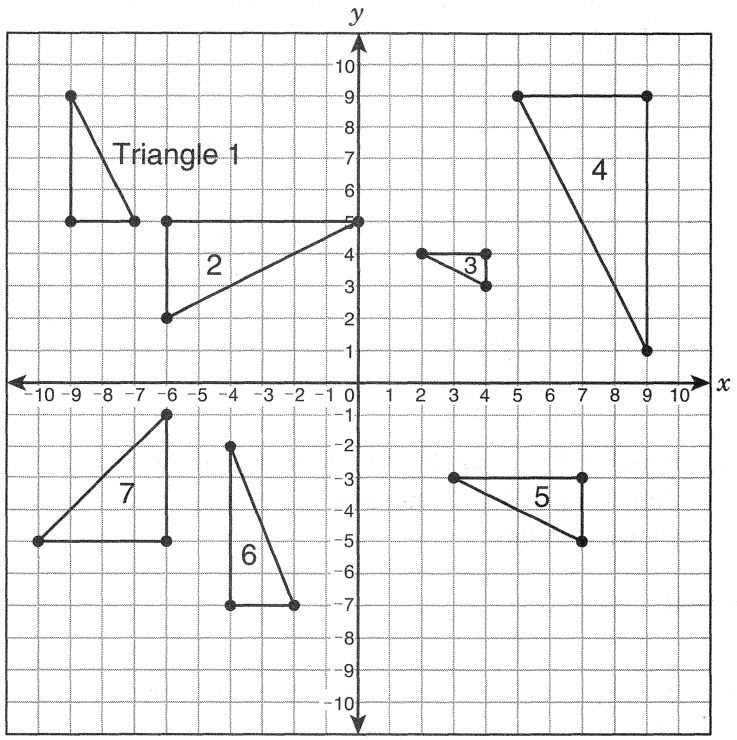
A 90° clockwise rotation

B dilation by a scale factor of $\frac{1}{2}$

C dilation by a scale factor of 2

D translation 2 units up

5 Look at each triangle on the grid below.



Which triangle is **not** similar to triangle 1?

A triangle 3

B triangle 4

C triangle 5

D triangle 6

6 Raul performs three transformations on a figure to produce an image that is similar but not congruent to the pre-image. Which type of transformation must have been one of the three?

A dilation

B reflection

C rotation

D translation

7 At 5:00 p.m. on a sunny day, a person 5 feet tall casts a shadow 7 feet long. At the same time, a tree nearby casts a shadow 56 feet long. What is the height of the tree, in feet?

A 40

B 45

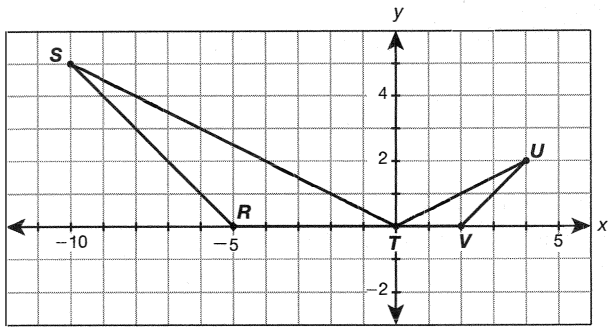
C 54

D 77

8 Rectangle A is transformed in the coordinate plane to map onto Rectangle A'. How can you use the corresponding coordinates of the vertices of the two rectangles to determine if the figures are similar?

Explain your answer.

9 Triangles *VUT* and *RST* are shown on the coordinate plane below.



$\triangle RST$ is similar to $\triangle VUT$. What is a sequence of transformations that would produce $\triangle RST$ from $\triangle VUT$?

Explain your answer.

EXIT TICKET

NY-8.G.4

Now that you have mastered dilations and similarity, let's solve the problem in the **Real-World Connection.**

Ekani is going to paint a design on her bedroom wall. She projects a coordinate grid onto the wall to help her. If she begins with the polygon labeled Figure *A*, how can she transform the figure to create Figure *A*′?